Math 10A with Professor Stankova
Quiz 7; Wednesday, 10/11/2017
Section #107; Time: 11 AM
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Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False For $f(x)$ continuous on $[a,b]$, the function $F(x) = \int_a^x f(u)du$ is the unique anti-derivative of f on $[a,b]$ satisfying $F(a) = 0$.

2. **TRUE** False If $f(x)$ is continuous on $[a,b]$, then $F(x) = \int_a^x f(u)du$ is continuous on (a,b).

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) Write $\int_2^5 \sqrt{1+\sqrt{x}}dx$ as a limit of right endpoint Riemann sums (no need to calculate it).

Solution: The interval $[2,5]$ has length 3 so each subinterval will have length $\frac{3}{n}$. These subintervals are $[2, 2+3/n], [2+3/n, 2+6/n], \ldots, [5-3/n, 5]$. Using the right endpoint method, we have

$$\int_2^5 \sqrt{1+\sqrt{x}}dx = \lim_{n\to\infty} R_n$$

$$= \lim_{n\to\infty}\left[\sqrt{1+\sqrt{2+\frac{3}{n}}}\cdot\frac{3}{n} + \sqrt{1+\sqrt{2+\frac{6}{n}}}\cdot\frac{3}{n} + \cdots + \sqrt{1+\sqrt{5}}\cdot\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\sqrt{1+\sqrt{2+\frac{3i}{n}}}\cdot\frac{3}{n}.$$

(b) (7 points) Find $\int \sqrt{1+\sqrt{x}}dx$

Solution: We guess $u = \sqrt{1+\sqrt{x}}$ and so $u^2 = 1+\sqrt{x}$ and $x = (u^2-1)^2 = u^4 - 2u^2 + 1$. So we have that $dx = (4u^3 - 4u)du$ and so

$$\int \sqrt{1+\sqrt{x}}dx = \int u(4u^3 - 4u)du = \int 4u^4 - 4u^2 du.$$

$$= \frac{4u^5}{5} - \frac{4u^3}{3} + C = \frac{4(1+\sqrt{x})^{5/2}}{5} - \frac{4(1+\sqrt{x})^{3/2}}{3} + C.$$